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ONLINE LEARNING FOR STRUCTURAL KERNELS

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Abstract

While online learning techniques have existed since Rosenblatt's introduction of the Perceptron in 1957, there has been a renewed interest lately due to the need for efficient classification algorithms. Additionally, kernel techniques have allowed online learning to be extended to problems whose classes are not linearly separable in their native space. Online algorithms typically result in poorer performance than support vector machines, but they have the advantage of vastly reduced computational resources and in some cases do offer performance comparable to the state of the art.

This work provides an implementation of several state of the art online learning algorithms as part of the SVM-Light-TK software package, and provides several evaluations of the same. Novelties include the evaluation of such algorithms in the face of high-dimensional structural kernels, and their application to Question Classification and Semantic Role Labelling Boundary Classification tasks from the NLP domain. Additionally, we present a comparison of these online algorithms with batch mode Support Vector Machines.

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Declaration

I declare that this thesis was composed by myself, that the work contained herein is my own except where explicitly stated otherwise in the text, and that this work has not been submitted for any other degree or professional qualification except as specified.

Patrick McKenna

To my dad

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Chapter 1

Introduction

1.1 Problem Statement

In an age when the sheer amount of data requiring processing is ever increasing, it is important to examine methods that are capable of coping. Since its introduction to the field of machine learning, the Support Vector Machine (SVM) [Boser, Guyon & Vapnik, 1992; Cortes & Vapnik, 1995] has become an indispensable tool for solving binary classification problems. However, SVMs operate in an 'offline' or 'batch-learning' framework where the predictive function is fixed after the initial training. This technique works extremely well when the probability distributions from which the training and test examples are drawn are identical, but this cannot always be guaranteed. Additionally, in order to optimize the model, the SVM algorithm must hold all the training examples in memory and revisit them multiple times resulting in a relatively long learning time. It is for these reasons that interest in online learning algorithms has been renewed.

1.2 Background

While it was first introduced as a model for information storage in the brain, Rosenblatt's Perceptron is perhaps the first example of a linear classifier trained with supervised learning in the online setting. This class of algorithms lost popularity following the publication of Minsky & Papert [1969] which lamented their limitation to linearly separable problems, the XOR function given as an example.

Linearly separable problems can sometimes be quite rare; all it can take is

a single mislabeled example (an unfortunately common occurrence) or outlier to invalidate many algorithm's convergence guarantees. While multi-layer perceptron networks were later shown to overcome this limitation, that is not the route that this thesis will examine. When it was first introduced, the Support Vector Machine [Boser et al., 1992] was another example of a linear classifier which had this limitation, but with later advances [Cortes & Vapnik, 1995] was able to overcome it.[1]

While an SVM is defined as a linear classifier, the use of kernel methods allows for the mapping of examples into another space where they can be compared and found to be (hopefully) linearly separable. The so called "kernel trick" removes the need for explicitly storing an example's kernel-space representation, as we only rely on the distances between examples in kernel-space. This implicit representation allows for kernels to leverage the power of a potentially infinite set of dimensions in their attempt to define a space where the data is separable by a hyperplane.

Even so, kernalized SVMs could still be thwarted by mislabelled data which can render a problem inseparable in any space.Soft-margin SVMs [Cortes & Vapnik, 1995] permit examples to lie within the margin and even stray to the other side of the separating hyperplane meaning they will be misclassified by the model, as a tradeoff for a wider margin. Even in a linearly separable case, the inclusion of soft-margin to the SVM optimization problem can allow for a wider margin typically resulting in improved generalization.

"Although SVM was initially stated as a batch-learning technique, it significantly influenced the development of kernel methods in the online-learning setting"[Dekel, Shalev-Shwartz & Singer, 2008]. The kernalized Perceptron algorithm lead the way for many other online variants and it is these which we examine in this thesis.

This discrepancy in their treatment of data is one of the major differences between SVMs and the online algorithms. In the batch setting, we are typically provided two sets: a training set used to fix the model, and a test set used to evaluate it. In the online setting however, there need not be such a clear distinction between the two sets. Every example is first presented without a label and the model's prediction is computed and output. The true label is then revealed to the algorithm and it has the opportunity to update its model before waiting for the next example. Unfortunately this discrepancy leads to

[1]Both the Perceptron and SVM algorithms can be applied to regression problems, this thesis only examines their use for classification.

difficulty in directly comparing online and batch methods.

Online to batch conversion techniques have been put forward [Helmbold & Warmuth, 1995], however it is understood that any given online algorithm will be less accurate than a well designed batch algorithm. Such conversions are only really applicable in situations where the advantages of online methods are unnecessary. Where they would really excel would be in situations where batch mode algorithms could not be directly applied, hence the difficulty in their comparison.

Convolution kernels, such as the Tree Kernel used in natural language processing (NLP) tasks are unique in that they are recursive kernels over structured data. They lead to exponentially large feature spaces in which all substructures are compared. For many tasks in this domain, such as the semantic role labelling task, we are presented with vast numbers of example instances. In such cases it would be useful to have reliable techniques that are capable of processing the entire training set, a challenge that SVMs cannot always meet.

1.3 Objectives

SVM-Light [Joachims, 1999] is a highly optimized and widely used implementation of Support Vector Machines written in C. It has been extended into the NLP domain by the addition of a number of Convolution Kernels including the Tree Kernel of [Collins & Duffy, 2002a] as SVM-Light-TK [Moschitti, 2006]. This thesis extends this software with implementations of several state of the art online learning algorithms and provides a thorough comparison of their performances in benchmark machine learning tasks as well as problems from the NLP domain.

1.4 Scope

This thesis is primarily concerned with the evaluation and comparison of online learning algorithms. While this thesis does make use of different kernels depending on the dataset, it is by no means intended as a comparison of the kernels' efficacy.

1.5 Contributions

The contributions made by this thesis to the scientific community include the implementation of state of the art online learning algorithms as part of the SVM-Light-TK software package. The results of their comparative evaluations is particularly novel in that their performance was also compared to batch mode Support Vector Machines.

Additionally, the application of online learning algorithms to NLP domain problems with structural data, and the evaluation of their performance when using Convolution kernels is also novel. The behaviour of single-pass online algorithms in such an exponential feature space was previously unknown.

1.6 Structure

This thesis is organized as follows. Chapter 2 introduces fundamental concepts, techniques and the notation used throughout the work. Chapter 3 presents the theory behind several state of the art online learning algorithms, while Chapter 4 describes the experimental framework used to compare them. The experiments and results using traditional feature-vector kernels are presented in Chapter 5, while those examining structural kernels appear in Chapter 6. We conclude with a discussion of the results in Chapter 7.

Chapter 2

Introduction to Concepts and Related Works

2.1 Binary Classification

Unless otherwise stated, the class of machine learning problem addressed in this thesis is that of binary classification. We are provided with a set of data instances which we assume are points $\mathbf{x} \in \mathbb{R}^n$. Each instance has a corresponding label $y \in \{-1, +1\}$.

The goal of a binary classification algorithm is to learn a function with which it can correctly predict the class of unlabelled examples. The predicted label is assigned according to which side of the separating hyperplane the instance falls and is typically denoted as $\hat{y}$. However, $\hat{y}$ can also refer to the distance from the hyperplane and whose magnitude is indicative of the confidence of the prediction. In such cases $\text{sign}(\hat{y})$ should be understood as the predicted class.

The equation of a hyperplane is

$$f(\mathbf{x}) = \mathbf{w} \cdot \mathbf{x} + b = 0 \qquad \mathbf{x}, \mathbf{w} \in \mathbb{R}^n \qquad b \in \mathbb{R},$$

where $\mathbf{x}$ is the vector representing the example being classified, and $\mathbf{w}$ is a vector of weights which are set by the learning algorithm (geometrically, it is interpreted as the gradient of the hyperplane). The inner product between the weight vector and the input vector is a weighted sum and is added to the bias term b. The classification function is $h(\mathbf{x}) = \text{sign}(f(\mathbf{x}))$. This is referred to as the primal form.

One can also represent the hyperplane by it's dual form as

$$f(\mathbf{x}) = \sum_i \alpha_i y_i \mathbf{x}_i \cdot \mathbf{x} + b.$$

The dual form represents the weight vector implicitly as a linear combination of previously observed instances weighted by α. Exactly which instances are stored and the value assigned to α varies between algorithms and will be examined further in Chapter 3. This implicit representation is essential to the kernel methods discussed in the following sections, and also for the algorithms in the online setting.

2.2 Support Vector Machines

The Support Vector Machine is a supervised learning algorithm for optimizing a binary classifier. It works by finding the hyperplane which separates the instances into their two classes. Geometrically, the space between the instances are separated by a margin, and the SVM attempts to find the hyperplane with the maximum margin. See Figure 2.1 for an illustration of this concept.

2.2.1 Hard Margin SVM

The functional margin of an example with respect to a hyperplane is $\gamma = y(\mathbf{w}\cdot\mathbf{x}+b)$. When we have a set of examples, we can observe the distribution of functional margins with respect to a hyperplane, the minimum of which is known as the functional margin of the hyperplane. The goal of the Support Vector Machines (SVMs) is to find the hyperplane with the largest functional margin with respect to the example instances in the training set. This hyperplane is known as the maximum margin classifier.

The maximum margin hyperplane can be found using the optimization problem:

$$\text{minimize } \frac{1}{2}\|\mathbf{w}\|^2 \qquad \text{subject to } y_i\left(\mathbf{w}\cdot\mathbf{x}_i + b\right) \geq 1, i = 1, \ldots, m$$

where $\|\mathbf{w}\|$ is the euclidean norm.

The solution to which can be expressed as a linear combination of example instances $\mathbf{w} = \sum_i \alpha_i y_i \mathbf{x}_i$. In the case of the optimal hyperplane, only those instances which lie on the margin will have α_i values different from zero, and it is these instances which are the support vectors.

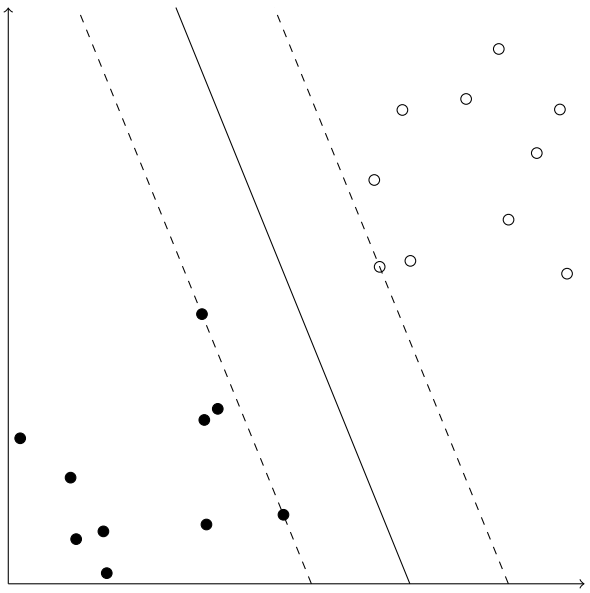


Figure 2.1: The maximum margin classifier for a set of linearly separable training data. The set of points which lie on the margin are referred to as the support vectors.
Note: This figure depicts the geometric margin of the hyperplane and assumes that the weight vector and bias term are normalized.

2.2.2 Soft Margin SVM

While the above formulation is sufficient when the classes are linearly separable, this is generally not the case due to effects such as label noise. Even when the data is linearly separable, such a solution may not be desirable as it can lead to overfitting the training data. The Soft Margin method was introduced by Cortes & Vapnik [1995] to handle these common situations.

They introduce a set of slack variables ξ_i which measure the distance from any misclassified instances to the decision boundary hyperplane. The relaxed condition for the optimal hyperplane is

$$y_i(\mathbf{w} \cdot \mathbf{x}_i + b) \geq 1 - \xi_i \qquad \forall x_i \text{ where } \xi_i \geq 0.$$

The objective function is extended to penalize the incorrect classification of examples, and the parameter C is introduced to control the tradeoff between maximizing the margin and minimizing the training error. As C approaches infinity, the solution approaches that of the hard margin SVM and the number of training errors decreases to zero. When C is smaller, the solution can account for some misclassified instances using the slack variables and get a wider margin which models the rest of the data well. If C is zero however, we obtain the trivially optimal $\|\mathbf{w}\| = 0$ as the problem then allows for infinite slack. The maximum soft-margin hyperplane is described by the following optimization problem:

$$\text{minimize } \frac{1}{2}\|\mathbf{w}\|^2 + C\sum_i \xi_i \quad \text{ subject to } y_i\left(\mathbf{w} \cdot \mathbf{x}_i + b\right) \geq 1, i = 1, \ldots, m.$$

It is important to note however that this formalization does not directly minimize the number of errors, but rather the sum of the distances from the hyperplane.

Parameters One can also include a cost factor J "by which training errors on positive examples outweigh errors on negative examples" [Morik, Brockhausen & Joachims, 1999]. This is important to account for biases in the data or situations where the penalties associated with false positive and false negative predictions differ (such as medical diagnosis).

$$\text{minimize } \frac{1}{2}\|\mathbf{w}\|^2 + C\left(J\sum_i \xi_i^+ + \sum_i \xi_i^-\right)$$

Dual representation The soft-margin SVM optimization problem can also be formulated in the dual representation discussed in section 2.1. Find $\alpha_1, \ldots, \alpha_m$

such that

$$\sum_{i=1}^{m} \alpha_i - \frac{1}{2} \sum_{i,j=1}^{m} y_i y_j \alpha_i \alpha_j \left(\mathbf{x}_i \cdot \mathbf{x}_j + \frac{1}{C} \delta_{ij} \right)$$

is maximized and

$$\alpha_i \geq 0, \forall i = 1, \ldots m \qquad \sum_{i=1}^{m} y_i \alpha_i = 0$$

It is this dual representation that allowed for the adoption of kernel methods as discussed below.

2.3 Kernel Methods

By examining the SVM dual problem, it can be seen that the gradient of the hyperplane $\mathbf{w}$ is stored implicitly as a linear combination of inner products between support vectors.

We call a function K on two objects a kernel if it can be shown that it is an inner product in some feature space $\mathcal{H}$. We write the function mapping a classifiable object o into the feature space vector $\mathbf{x}$ as $\phi(o) = \mathbf{x}$.

$$f(o) = \mathbf{w} \times \phi(o) + b = 0$$

The so called 'kernel trick' lies in the fact that we can rewrite the hyperplane defining the decision boundary as

$$f(\mathbf{x}) = \left(\sum_i y_i \alpha_i \phi(o_i) \right) \cdot \phi(o) + b = \sum_i y_i \alpha_i \phi(o_i) \cdot \phi(o) + b$$

The kernel function associated with the mapping function ϕ is

$$K(o_i, o) = \langle \phi(o_i) \cdot \phi(o) \rangle$$

and so we can compute the classification of any instance without having to apply the mapping ϕ directly.

$$f(\mathbf{x}) = \sum_i y_i \alpha_i K(o_i, o) + b$$

This allows for the use of kernels with potentially infinite dimensional implicit feature spaces.

2.4 Online Learning

While online learning techniques have been studied since the introduction of the Perceptron in 1958, they suffered a loss of popularity due to Minsky's publication criticizing that algorithm's inability to classify data which is not linearly separable [Minsky & Papert, 1969]. The kernel trick first published by Aizerman, Braverman & Rozoner [1964] has been applied to the perceptron algorithm in its dual form giving it the power to solve problems that are not linearly separable in their native space.

The training of a support vector machine is at least quadratic in the number of examples. Attempts have been made to create mixtures of SVMs for large scale problems; splitting the data into subsets allow for a collection of SVMs to be trained in parallel [Collobert, Bengio & Bengio, 2002]. Interest in online learning methods have also been renewed to confront these types of problems, due to their near state-of-art performance and greatly reduced computational costs.

In the batch setting we are given two sets: a training set used to find a good model, and a test set for evaluation. A batch mode algorithm will enumerate over the training set (typically more than once) and fix the weight vector. This model will then be used to evaluate on the test set. Batch methods often have very good formal guarantees if the instances in the training and test sets are drawn from the same fixed (but unknown)distribution.

In the online setting, no distinction is made between training and test sets of instances. Learning is performed in rounds, where a single unlabelled instance $\mathbf{x}$ is presented to the algorithm. The online algorithm must output its prediction of the label $\hat{y}$ before receiving the true label y. Learning proceeds incrementally in this way until the instances are exhausted.

Algorithm 1 Online Framework

initialize classifier $f_1(x)$
for $t = 1 \dots T$ **do**
    $x_t \leftarrow$ input instance
    $\hat{y}_t = f_t(x_t)$ prediction
    $y_t \leftarrow$ feedback label
    $\ell(\hat{y}_t, y_t)$ loss
    $f_{t+1} \leftarrow$ update prediction rule
end for

Because online algorithms only process one example at a time, they tend

to be fast and have a predictable runtime. Another benefit of their simplicity is that they are usually straightforward to implement. The framework is very adaptable; to define a new online learning algorithm one only needs to define prediction and loss functions, as well as an update rule. The goal of an online learning algorithm is to minimize the cumulative loss $\sum_t \ell(\hat{y}_t, y_t)$. One can also easily apply an online algorithm in the batch setting, while the reverse is not generally true.

These benefits aside, it is still generally understood that any given online algorithm will not be as performant as a well designed batch algorithm.

2.5 Structural Kernels

Bod [1998] introduced a representation for comparing trees which relied on comparing all subtrees. However, the set of subtrees grows exponentially with the size of the tree, and this leads to huge grammars and is computationally difficult to work with. The Tree Kernel however is capable of counting all common sub-structures using an implicit representation [Collins & Duffy, 2002a,b]. The inner product is computed in polynomial time relative to the size of the trees rather than the number of substructures which is exponential.

This is an example of a "Convolution Kernel" which performs 'a recursive calculation over the "parts" of a discrete structure' (in this case, parse trees), the premise of which was first introduced by Haussler [1999] and Watkins [2000].

> If one can construct kernels over the parts then one can combine these into a kernel over the whole object. [Collins & Duffy, 2002a]

This idea is extended recursively: kernels which compare the atomic parts of an object are combined in such a way that the final result retains information about the object's structure.

In the case of the Tree Kernel, the kernel space is of d dimensions where d is the number of tree fragments in the training data and is potentially infinite when used in an online setting with an unbounded number of trees. Each tree is represented by a vector where "the i'th component counts the number of occurrances of the i'th tree fragment." Let $h_i(T)$ be the number of occurrences of the i'th tree fragment in tree T, so T can be represented as $\mathbf{h}(T) = \langle h_1(T), h_2(T), \ldots, h_d(T) \rangle$.

As the dimensionality of this space is so large (a given tree has an exponential number of subtrees and the number of trees is potentially unbounded)

Collins and Duffy developed the Tree Kernel algorithm in such a way so that its computational complexity does not depend on d.

$$\begin{aligned} K(T_1, T_2) &= \mathbf{h}(T_1) \cdot \mathbf{h}(T_2) \\ &= \sum_{i}^{d} h_i(\mathbf{x}_1) h_i(\mathbf{x}_2) \\ &= \sum_{i}^{d} \left(\sum_{n_1 \in N_1} I_i(n_1) \right) \left(\sum_{n_2 \in N_2} I_i(n_2) \right) \\ &= \sum_{n_1 \in N_1} \sum_{n_2 \in N_2} \sum_{i}^{d} I_i(n_1) I_i(n_2) \end{aligned}$$

Where $I_i(n)$ is an indicator function equal to 1 if fragment i is left-most at node n, and $\Delta(n_1, n_2) = \sum_i I_i(n_1) I_i(n_2)$ counts the number of common subtrees rooted in nodes n_1 and n_2. This can be recursively defined as follows:

$$\Delta(n_1, n_2) = \begin{cases} 0 & \text{if } n_1 \neq n_2 \\ 1 & \text{if } n_1 = n_2 \text{ and are pre-terminals} \\ \prod_{j=1}^{nc(n_1)} (1 + \Delta(ch(n_1, j), ch(n_2, j))) & \text{otherwise.} \end{cases}$$

Where $nc(n)$ is the number of children of n, and since the productions at n_1 and n_2 are the same, $nc(n_1) = nc(n_2)$. Additionally, the function $ch(n, j)$ gives the j'th child of node n.

Collin's and Duffy also introduce a λ parameter ($0 < \lambda \leq 1$) to reduce the weight of larger tree fragments by augmenting each case with a multiplication with λ. The recursive calls result in smaller weights for larger trees, effectively normalizing the output.

Another suggested method for normalizing the effect of the size of tree fragments is to compute the following kernel:

$$K'(T_1, T_2) = \frac{K(T_1, T_2)}{\sqrt{K(T_1, T_1) K(T_2, T_2)}}$$

Chapter 3

State of the Art Online Learning Algorithms

3.1 Unbounded Algorithms

3.1.1 Perceptron

While originally introduced as a model of information storage in the brain, the Perceptron algorithm is a very simple linear classifier which naturally fits into the online framework. It functions by storing a hypothesis vector $\mathbf{w}$ which is initialized to zero. When presented with an unlabelled instance $\mathbf{x}$, it predicts its class $\hat{y}$ as the sign of the scalar product of the instance with the hypothesis vector. When the actual label y is revealed, if it differs from the prediction, the hypothesis vector is updated to include the misclassified instance $\mathbf{w} = \mathbf{w} + y\mathbf{x}$. If the prediction was correct, the hypothesis remains unchanged. This process is repeated for each example.

$$\hat{y} = \text{sign}(\mathbf{w} \cdot \mathbf{x})$$

$$\mathbf{w} = \sum_{i=0}^{m} y\mathbf{x}_i$$

The Perceptron can be easily adapted to a batch learning framework by running the algorithm repeatedly through the training set. This is typically done until it finds a hypothesis vector which correctly classifies the entire training set, which is then used to predict the labels of the instances in the test set.

Block (1962), Novikoff (1962) and Minsky and Papert (1969) have shown that if the data are linearly separable, then the percep-

tron algorithm will make a finite number of mistakes, and therefore, if repeatedly cycled through the training set, will converge to a vector which correctly classifies all of the examples. Moreover, the number of mistakes is upper bounded by a function of the gap between the positive and negative examples, a fact that will be central to our analysis. [Freund & Schapire, 1999]

Algorithm 2 Perceptron

$\mathbf{w}_0 \leftarrow \mathbf{0}; b_0 \leftarrow 0; k \leftarrow 0; R \leftarrow \max_{1 \leq i \leq T} \|\mathbf{x}_i\|$
for $i = 1 \ldots T$ **do**
  if $y_i \left(\mathbf{w}_k \cdot \mathbf{x}_i + b_k\right) \leq 0$ **then**
    $\mathbf{w}_{k+1} = \mathbf{w}_k + \eta y_i \mathbf{x}_i$
    $b_{k+1} = b_k + \eta y_i R^2$
    $k = k + 1$
  end if
end for

The kernel methods developed for use in Support Vector Machines [Boser et al., 1992] were actually a continuation of the work done by Aizerman et al. [1964] of combining kernel functions with the Perceptron algorithm. They noted that in the dual form where the hypothesis vector is represented by a set of instances and weights, all the computations involving instances can be formulated using inner products between them. As we have seen in Section 2.3, we can then use a valid kernel instead of the inner product in the native space, and the Perceptron's performance is now only dependent on linear separability in the kernel space.

Algorithm 3 Perceptron – Dual Form

$\mathbf{w} = 0; \alpha \leftarrow 0; R \leftarrow \max_{1 \leq i \leq T} \|\mathbf{x}_i\|$
for $i = 1 \ldots T$ **do**
  if $y_i \left(\sum_{j=1 \ldots i} \alpha_j y_j \mathbf{x}_j \cdot \mathbf{x}_i + b\right) \leq 0$ **then**
    $\alpha_j = \alpha_j + \eta$
    $\beta = \beta + \eta y_i R^2$
  end if
end for

3.1.2 Averaged Perceptron

The main idea behind the Voted and Averaged Perceptron algorithms developed by Freund & Schapire [1999] is that by averaging the output of many models we protect against overfitting. The algorithm maintains a history of the prediction vectors $\mathbf{v}$ generated after every mistake, the number of mistaken predictions made so far (and equivalently, the number of vectors stored in the model) is denoted by k. Additionally, the number of iterations this prediction vector "survives" without making a mistake is counted and used as that vector's weight in the majority vote. The intuition being that the number of correct classifications is indicative of the quality of the vector and "better" vectors have a larger weight in the final decision.

While Freund & Schapire describe their algorithm in the batch setting as a "deterministic leave-one-out conversion of the online perception algorithm," the bounds proven for the voted-perceptron's performance apply when the number of training epochs is one. In other words, after seeing every training example only once, or equivalently: the online setting.

Voted Perceptron Let $\mathbf{v}_i$ be the prediction vector computed for mistake i, and c_i be its weight (the number of examples presented since the last misclassification). Then the output prediction of the voted-perceptron when presented with an instance $\mathbf{x}$ is

$$\hat{y} = \sum_{i=1}^{k} c_i \operatorname{sign}\left(\mathbf{v}_i \cdot \mathbf{x}\right),$$

where the inner product can be computed as

$$\mathbf{v}_k \cdot \mathbf{x} = \sum_{j=1}^{k-1} y_{i_j} \left(\mathbf{x}_{i_j} \cdot \mathbf{x}\right),$$

allowing the storage of $\mathbf{v}$ in an implicit form as a set of instances. Furthermore, it can be computed incrementally taking advantage of the relation $\mathbf{v}_{j+1} \cdot \mathbf{x} = \mathbf{v}_j \cdot \mathbf{x} + y_{i_j}\left(\mathbf{x}_{i_j} \cdot \mathbf{x}\right)$, and use a kernel function K by replacing $\mathbf{x}_{i_j} \cdot \mathbf{x}$ with $K\left(\mathbf{x}_{i_j}, \mathbf{x}\right)$. By using this dual form, we can obtain the history of prediction vectors from the set of stored instances. The Perceptron model need only be augmented by the vote count c_i for each stored instance.

The update step however, is very different from the standard Perceptron. An update occurs after every instance, not only for those which the prediction was wrong. The output of the most recent prediction vector is compared with the true label. If correct the value of c_k is incremented, if not the new instance is added to the model.

Algorithm 4 Voted Perceptron

$\mathbf{w} = 0; \alpha \leftarrow 0; k \leftarrow 0$
for $i = 1 \dots T$ **do**
  // output voted prediction
  $\hat{y} = \sum_{j=1}^{k} c_j \operatorname{sign}(\mathbf{v}_j \cdot \mathbf{x}_i)$
  // update if most recent prediction vector is wrong
  if $y_i(\mathbf{v}_k \cdot \mathbf{x}_i) \leq 0$ **then**
    $\mathbf{v}_{k+1} = \mathbf{v}_k + y_i \mathbf{x}_i$
    $c_{k+1} = 1$
    $k = k + 1$
  else
    $c_k = c_k + 1$
  end if
end for

While the bias term is notably missing from their algorithm, this problem is sidestepped by the use of a polynomial kernel which augments the feature space to allow for finding hyperplanes which do not intersect with the origin.

$$K(\mathbf{a}, \mathbf{b}) = (1 + \mathbf{a} \cdot \mathbf{b})^d$$

Averaged Perceptron The Averaged Perceptron [Freund & Schapire, 1999] functions in the same manner as the Voted Perceptron, and varies only in how the weighted prediction is computed. Rather than counting the number of positive or negative predictions, the predictions are additionally weighted by their confidence.

$$\hat{y} = \operatorname{sign}\left(\sum_{i=1}^{k} c_i (\mathbf{v}_i \cdot \mathbf{x})\right)$$

Freund and Schapire also present a normalized version of the Averaged Perceptron,

$$\hat{y} = \operatorname{sign}\left(\sum_{i=1}^{k} c_i \left(\frac{\mathbf{v}_i \cdot \mathbf{x}}{||\mathbf{v}_i||}\right)\right)$$

However, they note that the performance difference between the two methods are always minor. In the experiments presented in this thesis will examine the performance of the unnormalized Averaged Perceptron variant.

Extensions Carvalho & Cohen [2006] explain how this averaging technique can be "trivially applied to any mistake-driven online algorithm" and do so.

Their experiments show favourable results when applying averaging to tasks outside of the Natural Language Processing domain, however within this domain this technique seems to reduce performance.

3.1.3 Passive Aggressive

The Passive Aggressive family of algorithms [Crammer, Dekel, Keshet, Shalev-Shwartz & Singer, 2006] are another example of mistake-driven additive-update online algorithms. However, it does not limit the definition of "mistake" to cases where an example was incorrectly labeled. The goal of the Passive Aggressive algorithm (PA) is not simply a positive margin, but a large one in order to predict with high confidence. It uses the following *hinge-loss* function,

$$\ell\left(\mathbf{w};(\mathbf{x},y)\right) = \begin{cases} 0 & y(\mathbf{w}\cdot\mathbf{x}) \geq 1 \\ 1 - y(\mathbf{w}\cdot\mathbf{x}) & \text{otherwise} \end{cases}.$$

Whenever the margin of a prediction is less than one (they note that this choice of threshold is "rather arbitrary"), the Passive Aggressive algorithm performs its update as follows,

$$\mathbf{w}_{t+1} = \mathbf{w}_t + \tau_t y_t \mathbf{x}_t.$$

The update rule must strike a balance between the amount of progress it makes and retaining the information previously learned. This tradeoff was first formalized by Helmbold, Kivinen & Warmuth [1999]. Progress, as Crammer et al. have defined it for this family of algorithms, is made by modifying the hypothesis such that it classifies the new instance with a sufficiently large margin.

Crammer et al. provide three different formulations for the value of τ_t. The first variant, called simply PA is defined as

$$\tau_t = \frac{\ell_t}{\|\mathbf{x}_t\|^2}.$$

This technique works well with linearly separable data. However, the occurrence of a mislabeled example almost guarantees that the problem is no longer linearly separable. By employing such an aggressive update step the PA algorithm makes itself susceptible to such label noise. When presented with a mislabeled example, the updated weight vector will likely cause prediction mistakes on subsequent examples. As such, PA-I and PA-II are extensions to the original algorithm to deal with this kind of problem.

PA-I and PA-II Inspired by Vapnik's soft-margin support vector machine classifiers, Crammer et al. introduce a slack parameter ξ to their objective function and an *aggressiveness parameter* C which controls its influence. In PA-I, the update function scales linearly with ξ, while in PA-II it scales quadratically.

$$\mathbf{w}_{t+1} = \arg\min_{\mathbf{w}\in\mathbb{R}^n} \frac{1}{2}\|\mathbf{w} - \mathbf{w}_t\|^2 + C\xi \quad \text{s.t. } \ell(\mathbf{w}; (\mathbf{x}_t, y_t)) \leq \xi \text{ and } \xi \geq 0 \quad \text{(PA-I)}$$

$$\mathbf{w}_{t+1} = \arg\min_{\mathbf{w}\in\mathbb{R}^n} \frac{1}{2}\|\mathbf{w} - \mathbf{w}_t\|^2 + C\xi^2 \quad \text{s.t. } \ell(\mathbf{w}; (\mathbf{x}_t, y_t)) \leq \xi \quad \text{(PA-II)}$$

The update rule for these two algorithms share the same closed form as PA with τ defined as follows,

$$\tau_t = \min\left\{C, \frac{\ell_t}{\|\mathbf{x}_t\|^2}\right\} \text{ (PA-I)}$$

$$\tau_t = \frac{\ell_t}{\|\mathbf{x}_t\|^2 + \frac{1}{2C}} \text{ (PA-II)}$$

By testing with synthetic data sets, they show that PA-I and PA-II are indeed more robust to both instance noise and label noise.

3.2 Budgeted Algorithms

The algorithms above are guaranteed to make only a finite number of mistakes if the data presented to them are linearly separable. However if this is not the case, given an infinite stream of data such algorithms will make an unbounded number of mistakes. Each such mistake results in another example being added to the model, resulting in a model of unbounded size.

Obviously this can be a problem as any physical implementation of these algorithms must run on a machine with finite memory. After processing a sufficiently large quantity of examples, the stored model will exhaust its available resources and the program will crash. This limitation is not in the spirit of online learning.

Furthermore, as the size of these models grow, so does the computational time needed to predict the class of each new unlabelled example. If their size is unbounded, new predictions will eventually grind to a halt.

A number of algorithms have adopted a fixed limit on the size of their model, typically referred to as the algorithm's "budget." The simplest of any such budgeted online algorithms is called the Stoptron, introduced by Orabona, Keshet & Caputo [2008] as a baseline budgeted algorithm. The Stoptron functions in

the exact same manner as the Perceptron, however once the budget is reached it simply stops storing new examples.

Other initial attempts at staying within the budget while still learning from new examples used various heuristics. For example, the Randomized Budget Perceptron [Cavallanti, Cesa-Bianchi & Gentile, 2007] begins discarding a support vector at random to make room for new examples once the budget is reached.

3.2.1 Forgetron

The forgetron is an attempt made at continuous learning by replacing the oldest examples when the budget B is reached. It uses the concept of shrinking the significance of every stored example at each step such that when one needs to remove an example, one can do so without significantly affecting the hypothesis vector. It was the first such budgeted algorithm to offer a relative mistake bound.

When a prediction mistake occurs, the instance is added to the model as in the Perceptron. An additional value σ is stored for every instance in the model, and initialized to 1 for newly added examples. After increasing the size of the model

$$I'_t = I_t \cup \{t\} \qquad\qquad f'_t = f_t + y_t K\left(\mathbf{x}_t, \cdot\right),$$

the shrinking coefficient ϕ is computed and the *shrinking step* applies it to every instance in the model

$$f''_t = \phi_t f'_t \qquad\qquad \sigma_{i,t+1} = \phi_t \sigma_{i,t}$$

reducing their effect on the hypothesis. Due to the recurring nature of this update, the oldest instances in the model will have the smallest value of σ and therefore the smallest effect on the hypothesis. After the shrinking step, if the algorithm is over budget the oldest stored instance is removed

$$I_{t+1} = I'_t \setminus \{r_t\} \qquad\qquad f_{t+1} = \sum_{i \in I_{t+1}} \sigma_{i,t+1} y_i K(\mathbf{x}_i, \cdot)$$

Forgetron For the simplest of their presented shrinking mechanisms they derive the constant U from the allocated budget B. This constant is used to bound the norm of the hyperplane function, which allows for their proof that the Forgetron is competitive with any hypothesis whose norm is also within these bounds. Their choice of ϕ for the Forgetron provides a balance between

attenuating the influence of older examples and the damage caused to the hypothesis.

$$\phi_t = \min\left\{(B+1)^{-\frac{1}{2(B+1)}}, \frac{U}{\|f_t'\|}\right\}$$

$$U = \frac{1}{4}\sqrt{\frac{(B+1)}{\log(B+1)}}$$

Self-Tuned Forgetron The Self-Tuned Forgetron augments the aforementioned Forgetron by computing the gentlest possible shrinking coefficient for each shrinking step. If the damage from the removal of the oldest example is small enough already, the Self-Tuned Forgetron chooses $\phi = 1$ and no shrinking occurs.

Dekel et al. make use of the function Ψ, defined as

$$\Psi(\lambda, \mu) = \lambda^2 + 2\lambda - 2\lambda\mu$$

to evaluate the penalty of removing the oldest instance of the model. Its value is accumulated over time in the variable Q, and compared against the number of mistakes made so far (M) to determine the optimal shrinking coefficient.

They use the term Ψ_t to indicate the damage caused of removing the oldest example at time t. If the budget is not yet full, $\Psi_t = 0$. Otherwise,

$$\Psi_t = \Psi(\sigma_{r_t,t+1}, y_{r_t} f_t''(\mathbf{x}_{r_t}))$$

where $\mathbf{x}_{r_t}$ is the instance slated for removal, and σ_{r_t} is its accumulated shrinking coefficients. Then we can compute the gentlest possible shrinking step as

$$\phi_t = \max\left\{\phi \in (0,1] : \Psi(\sigma_{r_t,t}\phi, y_r \phi f_t'(\mathbf{x}_{r_t})) + Q_t \leq \frac{15}{32} M_t\right\}.$$

Dekel et al. provide an analytical solution to this quadratic function, however it is excluded here for the sake of brevity.

Greedy Forgetron Prior to the shrinking step, we can examine the damage that would be caused by the immediate removal of an instance. If it is discovered that there exists an example in the active set whose removal prior to shrinking causes no more than $\frac{15}{32} M_t$ damage, then it can be safely removed without any shrinking. The following chooses such an instance if it exists, or the oldest example otherwise.

$$j = \arg\min i \in I_t \Psi(\sigma_{i,t}, y_i f_t'(\mathbf{x}_i))$$

$$r_t = \begin{cases} j & \text{if } \Psi\left(\sigma_{j,t}, y_j f'_t\left(\mathbf{x}_j\right)\right) \leq \frac{15}{32} \\ \min I_t & \text{otherwise} \end{cases}$$

The shrinking step is then performed as in the Self-Tuned Forgetron.

While this version of the algorithm is not limited to removing the oldest example of the model, Dekel et al. note that as the size of the stored model increases, the Greedy Forgetron tends to leave noisy examples in the model. As such, the literature treats the Self-Tuned Forgetron as the best algorithm from this family and it is this one which will be examined in Chapters 4 & 5.

3.2.2 Projectron

Orabona, Keshet & Caputo [2008] use a technique pioneered for the improvement of Support Vector Machine solutions by Downs, Gates & Masters [2001] to reduce the size of the support set (the set of active examples in the model) without changing the hypothesis. Rather than having a fixed budget however, the Projectron algorithm allows the user to set an input parameter that trades off between model sparseness and accuracy.

Upon making a mistake, if the new instance can be represented as a linear combination of the existing support set ($\mathbf{x}_t = \sum_{i=1}^{t-1} d_i \mathbf{x}_i$) it is not added to the model. Instead of adding it directly, the coefficients of the existing support set are updated to include the new instance, that is, $\alpha_i = y_i + y_t d_i$. If however, the new instance is linearly independent from the support set, it is added with $\alpha_t = y_t$ as usual.

While this technique does reduce the size of the support set without changing the hypothesis at all, it can still lead to infinitely large support sets given an infinite dimensional kernel, Orabona et al. employ the following solution by constructing two new hypotheses. The *temporal hypothesis* f'_t is simply the result of adding the instance to the support set as usual. The *projected hypothesis* f''_t is the projection of f'_t onto the space spanned by the previous support set S_{t-1}. If norm of the distance between the two hypotheses is below some threshold the projected hypothesis is adopted, otherwise the temporal hypothesis is used.

This threshold is referred to as the model sparseness parameter η. If η is equal to zero, the Projectron algorithm behaves exactly as the classical perceptron. As the value of η increases, the algorithm trades precision for sparseness. They show that their strategy ensures that the size of the support set is finite, regardless of the dimensionality of the kernel space. Their proof however, does not state that the size of the support set can be estimated before training.

Orabona et al. also prove a performance bound that shows the Projectron

is slightly worse than the Perceptron. If $\eta = 0$ they have the same bound, and its degradation is related to $\frac{1}{1-\eta\|g\|}$, where g is an arbitrary hypothesis.

The Projectron++ algorithm extends the Projectron by including an update step for when the new observation is correctly classified but only with a small margin. This mirrors the Passive-Aggressive family of algorithms improvements upon the Perceptron. However, the margin update step is only applied when the new instance can be projected onto the existing support set. A new instance correctly classified but with low confidence is never directly appended to the model (the "temporal hypothesis" as described for the error update step). While this would improve the classification rate, it does so by greatly increasing the size of the support set. The presented performance bound for the Projectron++ is closely related to that of PA-I, with degradation related to η. The Projectron++ algorithm is the first budgeted online learning algorithm to outperform the Perceptron and we examine its performance in the following Chapters.

Chapter 4

Method for the Empirical Comparison of Algorithms

4.1 Implementation Details

This work provides an implementation of several state of the art online learning algorithms as part of the SVM-Light-TK software package [Moschitti, 2006]. This library is written in the C programming language. Care was taken to write the learning algorithms in an object oriented fashion by using virtual function pointer tables to emulate classes. The strategy design pattern [Gamma, Helm, Johnson & Vlissides, 1995] was also followed to allow different algorithms to be substituted at runtime depending on the users request. To achieve this, each algorithm provides an implementation for the functions in the following code listing which the *svm_online.c* algorithm calls without knowledge of the underlying algorithm or its implementation.

Listing 4.1: Excerpt of *learning_algorithm.h* defining the common interface

```
// virtual functions
double predict_example (LearningAlgorithm*, DOC* doc);
void learn_example (LearningAlgorithm*, DOC* doc, double target);
void consolidate_model (LearningAlgorithm*);
void destroy (LearningAlgorithm*);

// static helper functions
void noop (LearningAlgorithm*);
LearningAlgorithm* algorithm_constructor(ONLINE_PARM*, MODEL*);
```

While some algorithms leverage advanced data structures during their online learning, the final models are consolidated into the format expected by SVM-Light-TK so that the *svm_classify* command can load them without further modification. The only exception is for storing the Voted and Averaged Perceptron models. In these cases, an integer for each instance in the model is needed to represent the number of votes they get during classification. This list of integers is appended to the end of the model file and *svm_classify* was modified to look for this extra data. If a list of votes is present, it calls the appropriate weighted classification function.

4.2 Hyperparameter Tuning

In order to select the optimal hyperparameters for a given learning algorithm, one typically performs a search over the possible values and evaluates the resulting models on a validation set (a subset of the training set). Attempting to ensure these evaluations generalize to the final test set, one can perform n-fold cross validation by evaluating n complimentary training and validation subsets. However, the hyperparameters for the algorithms under investigation in this thesis are not suited to this manner of tuning, as accuracy is trivially optimized at the expense of model size. For example, by setting the Projectron's sparsity parameter η equal to zero we obtain performance equivalent to the Perceptron (or similar to PA-I in the case of Projectron++). Increasing this value increases the model's sparsity and would thus lead to a decrease in prediction accuracy. Similarly, the aggressiveness parameter C behaves similarly to C in soft margin SVMs. Setting it to infinity reduces PA-I to the 'hard margin' Passive Aggressive algorithm, while setting it to zero allows for the trivially optimal $\|\mathbf{w}\|$. In practice, C has been shown to tradeoff between the learning rate and the final accuracy.

In the literature, these parameters are chosen somewhat arbitrarily. In an attempt to fairly compare algorithms, we elected to adopt the same values each algorithm's authors present. The aggressiveness parameter C for the PA-I algorithm was chosen to be equal to 1 as in [Orabona et al., 2008] which results in an update that is comparable to the basic Perceptron. For the Projectron++, η was set to 0.1 as in [Orabona, Keshet & Caputo, 2009].

4.3 Single-pass Evaluation

Due to the very nature of the online algorithms, the order in which the training data is presented has an impact on the resulting model. As such, all of the experiments presented herein were performed by training against five different permutations of the training set.

One of the strengths of bounded memory online learning algorithms is their applicability to continuous streams of data. When performing classification in realtime there is no clear separation between training and test sets, and as such no specific time when performing an additional pass over previous instances is appropriate.

4.4 Comparing Online to Batch Mode SVMs

The 'leave-one-out' method for the transformation of online learning algorithms to batch learning algorithms developed by Helmbold & Warmuth [1995] was adapted into the voted perceptron algorithm [Freund & Schapire, 1999].

While Freund & Schapire observe that using the Voted and Averaged Perceptron algorithms show improvements in performance after multiple passes through the training set (epochs), they "do not have a theoretical explanation for the improvement in performance following the first epoch."

Carvalho & Cohen [2006] found that this conversion technique (which was also applied to other online algorithms including Passive Aggressive I) improved performance when only performing a single pass through the training data, but this improvement was not apparent when the task was from the NLP domain. While novel, their study of single-pass online learning did not include the use of kernel functions. This thesis attempts to improve over their evaluation by applying structural kernels to NLP tasks where the classes have a better chance at being linearly separable, compared to using linear kernels over examples represented by bags-of-words.

4.5 Experimental Procedure

In this thesis we have chosen datasets with clearly defined training and test sets. In our experiments, the online algorithms are trained using a single pass over the training set. At this point, their models are fixed and their performance is evaluated over the test set. This procedure allows for the direct comparison

between single-pass online learning algorithms and batch-mode algorithms such as the SVM.

Additionally, we present recordings of the online algorithms' performance taken throughout the training stage. These plots show how the online learning algorithms behave as they are presented with novel instances, and are meant to reveal their behaviour in the setting of continuous data. As such, they are not directly comparable to batch-mode or multiple epoch algorithms.

4.6 Performance Measure

Rather than simply relying on Accuracy ($\frac{\text{correctly classified}}{\text{total examples}}$) which can be a deceptive metric when the number of positive and negative examples are skewed, this thesis presents its results using the F_1-measure which provides a balanced view of both precision and recall in the test set.

$$Precision = \frac{\text{true positives}}{\text{true positives} + \text{false positives}}$$

$$Recall = \frac{\text{true positives}}{\text{true positives} + \text{false negatives}}$$

$$F_1 = 2 \cdot \frac{Precision \cdot Recall}{Precision + Recall}$$

The sole exception to this is in the case of multiclassifiers which we examine in section 6.1. Since these models have more classes than simply positive and negative, the F_1 approach is not appropriate. For this reason we measure the multiclassifier's performance using its Accuracy.

Chapter 5

Feature-vector Kernel Experiments

For the experiments described in this chapter, two generic kernels were examined: an inhomogeneous polynomial kernel and the Gaussian radial basis function.

Polynomial Kernel

$$k(\mathbf{x}_i, \mathbf{x}_j) = (\mathbf{x}_i \cdot \mathbf{x}_j + 1)^d$$

The hyperparameter d sets the degree of the polynomial expansion. The implicit feature space obtained when using the polynomial kernel includes dimensions for each monomial up to degree d. This can be seen as a method of feature conjunction.

Gaussian radial basis function

$$\begin{aligned} k(\mathbf{x}_i, \mathbf{x}_j) &= \exp\left(-\gamma \|\mathbf{x}_i - \mathbf{x}_j\|^2\right) \\ &= \exp\left(-\gamma \left(\mathbf{x}_i \cdot \mathbf{x}_i - 2 \cdot \mathbf{x}_i \cdot \mathbf{x}_j + \mathbf{x}_j \cdot \mathbf{x}_j\right)\right) \end{aligned}$$

The implicit feature space corresponding to the Gaussian radial basis function is one of infinite dimensions. The hyperparameter γ (sometimes expressed as $\frac{1}{\sigma^2}$) controls the smoothness of the decision boundary and the area of influence of individual support vectors.

5.1 Synthetic Dataset

The synthetic datasets used in this thesis were constructed in the same manner as described in [Crammer et al., 2006; Dekel et al., 2008; Orabona et al., 2008, 2009]. Positive examples were sampled from a two-dimensional Gaussian distribution with a mean of $(1, 1)$ and the following covariance matrix:

$$\begin{bmatrix} 0.2 & 0 \\ 0 & 2 \end{bmatrix}$$

Negative examples were sampled from a Gaussian distribution with a mean of $(-1, -1)$ and the same covariance matrix. Label noise was added by flipping each label with probability $p = 0.1$.

Ten training sets of 10,000 instances each were generated along with a test set also containing 10,000 instances. Positive and negative instances were generated with equal probability. Each online algorithm was trained using a single pass over each of the training sets resulting in ten separate models. These resulting models were each tasked with predicting the labels of the test set without updating, the results of which were averaged and are presented below.

5.2 Synthetic Dataset Experimental Results

The test results after single-pass training are listed in Table 5.1. In it, we observe that the SVM results in the best performance while having a runtime orders of magnitudes greater than any of the online learning algorithms which treat the data with a single pass. The SVM model size is also quite large, second only to PA-I.

The averaged perceptron shows great improvement over the standard perceptron at this task, both in its F_1 score on the test set as well as requiring far fewer support vectors in the model.

The Projectron++ also managed to outperform PA-I, which is interesting as it is theoretically upper bounded by PA-I's performance. Also interesting is that the Projectron++ attains this level of performance with only 10 support vectors in the model. This probably speaks to the simplicity of the underlying data, but it demonstrates markedly more space efficient models than all the other examined algorithms. Furthermore, with such a small model, the runtime required to train the Projectron++ is vastly shorter than any other algorithm.

As expected, performance degrades as we restrict the size of the budget available to the Forgetron. These budget restrictions also decrease the runtime

Algorithm	F_1	Accuracy	SV	Runtime (s)
SVM	89.21 ± 0.25	89.19 ± 0.27	3054.80 ± 81.85	33.89 ± 3.19
Perceptron	73.45 ± 13.42	68.81 ± 14.53	2949.50 ± 706.23	1.86 ± 0.38
Averaged	88.99 ± 0.31	88.98 ± 0.32	1961.20 ± 51.99	1.28 ± 0.04
PA-I	76.00 ± 9.85	76.04 ± 8.41	3617.00 ± 93.16	2.24 ± 0.06
Projectron++	81.58 ± 13.36	81.11 ± 14.82	10.20 ± 0.42	0.03 ± 0.00
Forgetron_{1000}	65.90 ± 20.99	64.63 ± 12.14	1000	1.20 ± 0.07
Forgetron_{500}	53.26 ± 28.37	57.20 ± 9.37	500	0.69 ± 0.02

Table 5.1: Single-pass performance using polynomial kernel of degree 3 on the synthetic dataset. Online algorithms were trained on a single pass over the training dataset before being fixed and tested. This procedure was repeated using 10 random permutations of the data. The mean and standard deviation between the 10 models are listed. SV indicates the size of the support set of the final model after training. Runtime indicates the cpu-seconds required for training.

required for training, as we observe in the more extreme case of the Projectron++.

We also note that the Accuracy and F_1 measures are very similar in this example. This is due to the balance between positive and negative examples, an artifact of its construction as described in the previous section.

5.3 Adult Dataset

The Adult dataset from the UCI Machine Learning Repository consists of 32,561 training and 16,281 testing exampes. The task is to predict whether a given household has an income greater than $50,000.

Each datum provides 14 attributes of a household's census form, eight categorical attributes and six continuous. The continuous attributes have been discretized to result in a total of 123 binary attributes. This preprocessing was first performed by Platt [1999] and is available in SVM-Light format from `http://www.csie.ntu.edu.tw/~cjlin/libsvmtools/datasets/`.

As above, the algorithms were trained using a single pass over the training set before their models are fixed. Throughout the training however, we recorded the model's performance on the data instances it had been presented thus far.

For this dataset, a Gaussian radial basis function kernel was selected and tuned with $\gamma = 0.04$ ($\sigma^2 = 25$). This configuration was chosen in order to

Algorithm	F_1	Accuracy	SV	Runtime (s)
SVM	67.64 ± 0.02	83.36 ± 0.01	13056.60 ± 4.28	330.98 ± 6.20
Perceptron	51.24 ± 28.65	80.46 ± 3.22	6819.00 ± 33.08	73.56 ± 0.63
Averaged	42.87 ± 12.76	82.11 ± 1.84	6757.20 ± 51.91	76.30 ± 1.25
PA-I	51.50 ± 16.24	80.53 ± 3.31	12985.00 ± 46.44	134.87 ± 0.92
Projectron++	53.52 ± 13.45	79.60 ± 4.10	2009.80 ± 12.93	204.08 ± 3.19
Forgetron_{3000}	47.77 ± 17.66	74.07 ± 7.79	3000	41.12 ± 1.19
Forgetron_{1500}	34.38 ± 31.34	76.20 ± 4.12	1500	17.78 ± 0.17

Table 5.2: Single-pass performance using Gaussian kernel $\gamma = 0.04$ on the Adult dataset. Algorithms were trained on 5 different permutations of the training set before the models were fixed. SV indicates the size of the support set of the final model after training. Runtime indicates the cpu-seconds required for training.

match the kernel used in experiments performed in [Orabona et al., 2008], while providing a more in-depth look at the results including how the algorithms' performance progressed over time, and the complexity of the models produced.

In their study, Orabona et al. present the accuracy of the Projectron on this dataset. However, due to the un-balanced nature of this dataset (24% of the instances have positive labels), it is important to examine the F-measure as it takes both precision and recall into account. In order to train an SVM in this situation, an appropriate cost factor must be chosen to tune the weight of errors on positive examples versus errors on negative examples. Initial experimentation resulted in choosing a value of 2.5 for this hyperparameter.

5.4 Adult Dataset Experimental Results

Figure 5.1 plots the performance during the online pass through one of the five permutations of the training set. Similarly, figures 5.2 and 5.3 show the size of the model and the cumulative time elapsed during training respectively. The results of using the fixed models to predict the labels of the test set are presented in Table 5.2.

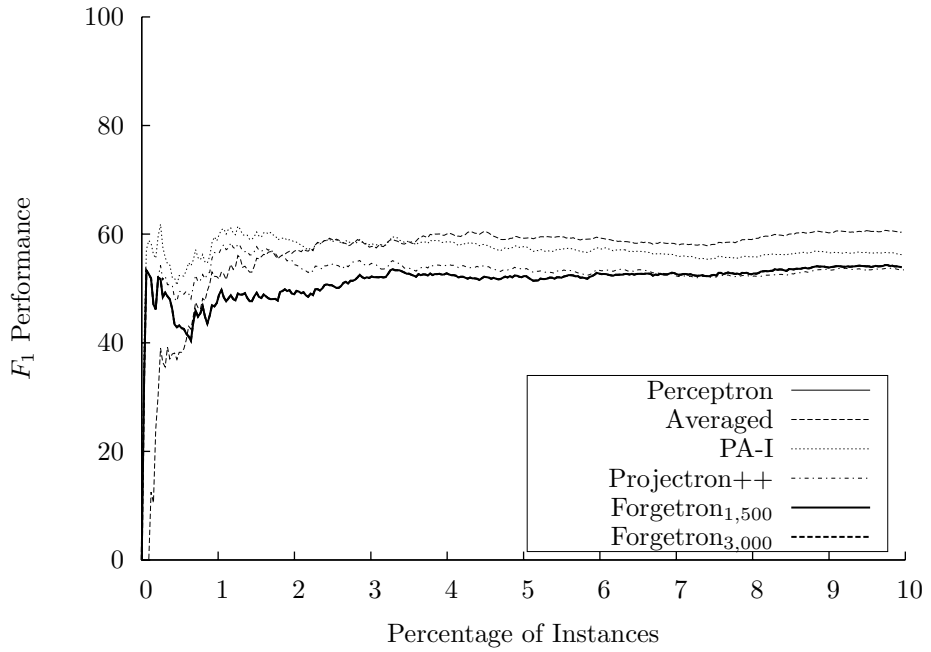
We observe in Table 5.2 that similarly to the synthetic dataset, the SVM demonstrates the highest performance at the expense of the longest runtime by far. PA-I stores almost twice as many support vectors as the Perceptron for little gain in performance. The Projectron++ algorithm however, demonstrates the best performance of all the online algorithms on this task and does so with far fewer support vectors in its model. While its test performance is good with a

very sparse model, is important to note however that its time complexity during training is even greater than PA-I, and approaches that of the SVM.

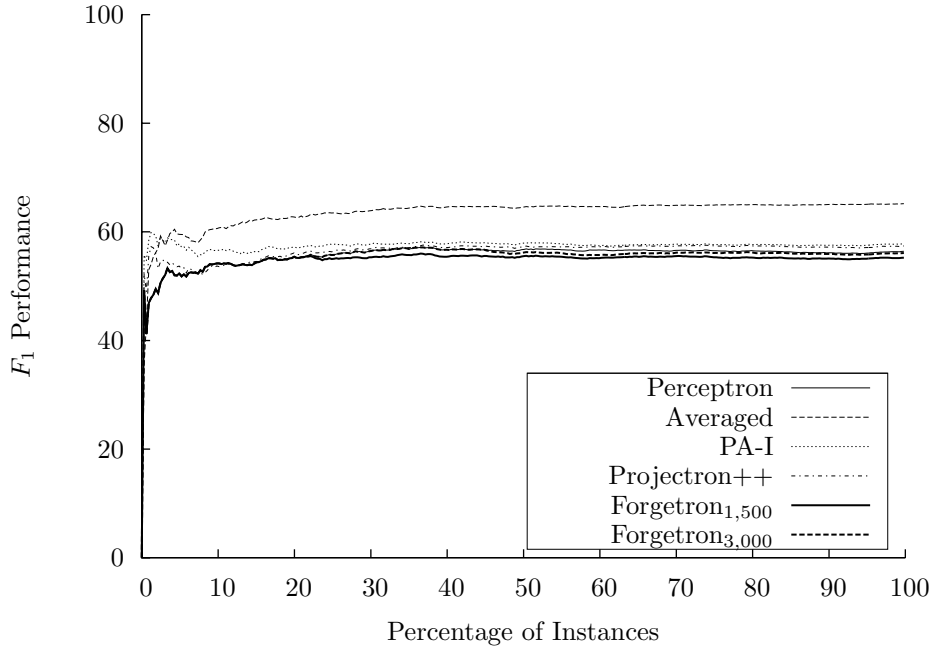
The Averaged Perceptron actually performs worse than the Perceptron in the final test, contrary to the suggestion by Carvalho & Cohen [2006] that voting improves performance in non-NLP tasks. Their study, however only examined the use of the inner product between instances (linear kernel).

During training however, the Averaged Perceptron is shown to be very performant in Figure 5.1. By making fewer mistakes than the Perceptron, it stores fewer support vectors and requires less computations to predict and update as reflected in Figures 5.2 and 5.3 respectively.

The budget cutoffs in the Forgetron algorithm can clearly be seen as the horizontal lines in Figure 5.2 and the transition to linear growth of time in Figure 5.3. It is at these points (roughly 20% and 45% of the training instances) where their performance diverges from the Perceptron in Figure 5.1, as expected. All the other online algorithms show polynomial growth in runtime. In descending order they are: Projectron++, PA-I, Perceptron, and Averaged.



(a) Online performance on first 10% of instances



(b) Online performance over all instances

Figure 5.1: Online performance during Adult9 training using a Gaussian kernel with $\gamma = 0.04$. Where 100% of the Instances corresponds to the set of 32,562 training instances. These learning curves show the cumulative performance of the algorithm as it processes a single permutation of the training dataset.

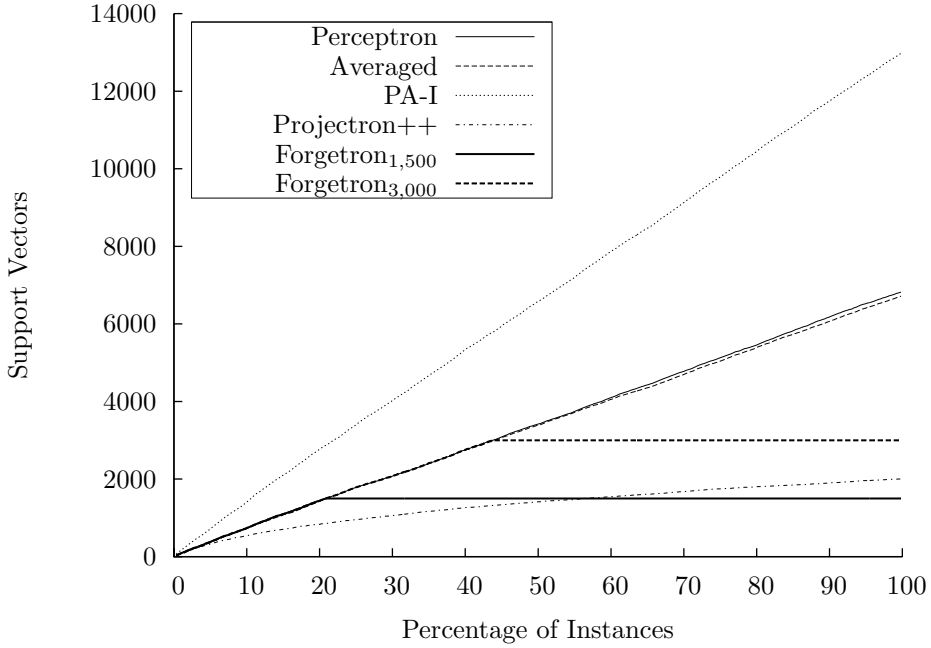


Figure 5.2: Growth of support set during Adult9 training using a Gaussian kernel with $\gamma = 0.04$. Where 100% of the Instances corresponds to the set of 32,562 training instances.

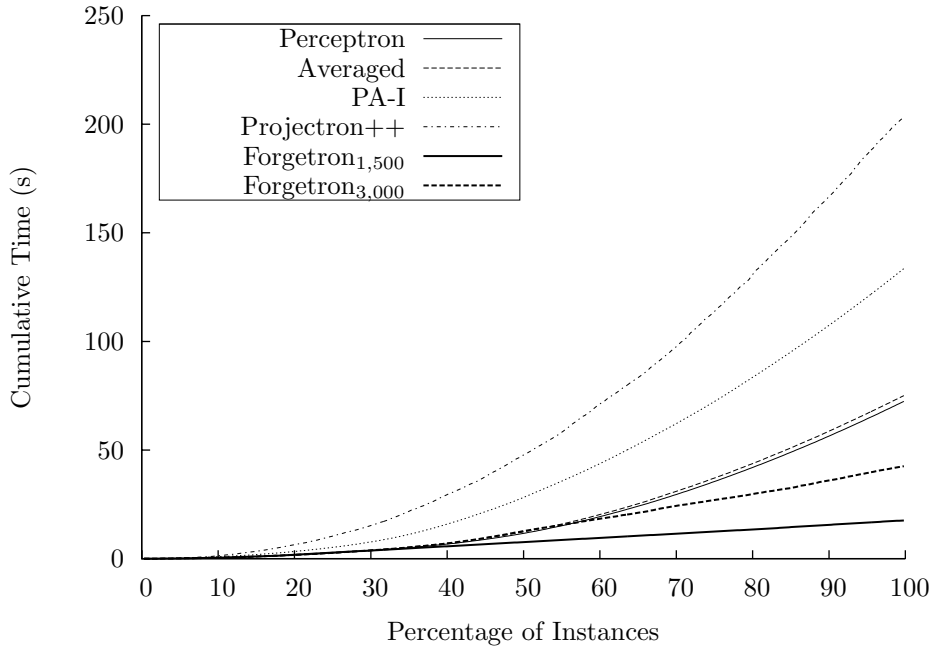


Figure 5.3: Cumulative runtime during Adult9 training using a Gaussian kernel with $\gamma = 0.04$. Where 100% of the Instances corresponds to the set of 32,562 training instances.

Chapter 6

Structural Kernel Experiments

The evaluation presented in this thesis is novel in its inclusion of structured kernels. The Syntactic Tree Kernel developed by Collins & Duffy [2002a] for evaluating natural language is applied to the Question Classification and Semantic Role Labeling tasks.

6.1 Question Classification

The question classification task is a multi-class decision problem, where each question belongs to one of 6 classes: *Abbreviation, Description, Entity, Human, Location*, or *Numeric*. While each of these classes have sub-classes such as date, money, or weight for the *Numeric* category, these have been discarded for these experiments.

The data for this problem consists of a training set of 5483 questions and a test set of 500 questions from `http://cogcomp.cs.illinois.edu/Data/QA/QC/`.

We implement the multi-classifier in the same manner as [Moschitti, Quarteroni, Basili & Manandhar, 2007] in order for our results to be directly comparable[1]. Initially one binary classifier is trained for each of the potential classes. The training data for each of these classifiers has been transformed in a ONE-vs-ALL scheme, where the one label for the desired class identifying positive

[1]While Orabona et al. [Orabona et al., 2009] provide a framework for structure learning using the Projectron algorithm, we elected to use the same setup for all of the examined algorithms

examples and all the remaining labels considered negative. This scheme has been used successfully for Question Classification in particular by Moschitti et al. [2007] and defended as a valid approach in general by Rifkin & Klautau [2004].

After training each class specific binary classifier using a single pass through the data and fixing the model, we can evaluate the algorithm's performance on the test set. Each test instance is presented to and classified by each of the 6 binary classifiers. The final output label is obtained by selecting the binary classifier with the greatest confidence in a positive classification. This training and testing procedure was performed five times using different permutations of the training data.

To compare against SVM, we present the results of both an untuned model as well as one with which the cost-factor values for each binary classifier were chosen according to the best results obtained by Moschitti et al. [2007]. This model is referred to as $\text{SVM}_{\text{tuned}}$.

6.2 Question Classification Experimental Results

Table 6.2 shows each algorithm's final performance on the multi-classification task, averaged between the five independent models trained on different permutations of the training set. Intermediate results can be seen in table 6.1 which shows the performance of each of the six binary classifiers.

While both the multi-class SVMs are clearly superior, this experiment shows the viability of obtaining performance approaching the state-of-the-art using single-pass online learning algorithms in both multi-class problems and in the NLP domain using structural kernels.

The strongest online model for the multi-classification task was created by the PA-I algorithm, followed by the Averaged Perceptron and the Projectron++. In some of the binary tasks we observe that the online algorithms occasionally outperform the untuned SVM (*Entity*, *Location*, and *Numeric* specifically.

	Abbreviation	Description	Entity	Human	Location	Numeric
$\text{SVM}_{\text{tuned}}$	84.21 ± 0.00	94.78 ± 0.00	80.43 ± 0.00	87.50 ± 0.00	82.58 ± 0.00	89.86 ± 0.00
SVM	80.00 ± 0.00	96.00 ± 0.00	63.89 ± 0.00	88.14 ± 0.00	77.61 ± 0.00	80.42 ± 0.00
Perceptron	72.13 ± 7.37	85.00 ± 10.60	54.46 ± 13.77	79.06 ± 10.46	67.50 ± 11.26	69.58 ± 8.06
Averaged	79.79 ± 5.69	93.31 ± 0.93	61.95 ± 4.69	87.08 ± 2.12	79.33 ± 3.00	79.84 ± 2.22
PA-I	79.79 ± 5.69	91.38 ± 1.99	67.04 ± 3.31	86.98 ± 1.09	76.57 ± 5.15	81.60 ± 3.50
Projectron++	76.92 ± 6.03	87.73 ± 1.49	63.31 ± 7.07	80.88 ± 5.36	80.39 ± 5.65	79.71 ± 5.44
Forgetron_{1000}	72.13 ± 7.37	85.00 ± 10.60	44.96 ± 14.66	79.06 ± 10.46	67.50 ± 11.26	69.58 ± 8.06
Forgetron_{750}	72.13 ± 7.37	73.71 ± 15.22	36.66 ± 8.44	66.44 ± 27.86	62.38 ± 11.02	69.58 ± 8.06
Forgetron_{500}	72.13 ± 7.37	47.91 ± 25.13	31.02 ± 11.96	61.2 ± 15.56	60.61 ± 12.30	59.49 ± 19.90

Table 6.1: Six binary classification tasks, one for each QC category. Each algorithm was presented with five permutations of the one-vs-all training sets resulting in five separate models for each binary task. After training the models are fixed. Their performance is given by the mean F_1 score of the five models on the test set.

Algorithm	Accuracy
$\text{SVM}_{\text{tuned}}$	89.00 ± 0.01
SVM	86.23 ± 0.08
Perceptron	73.47 ± 4.19
Averaged	82.76 ± 0.95
PA-I	83.43 ± 2.76
Projectron++	81.75 ± 2.15
Forgetron_{1000}	72.83 ± 3.80
Forgetron_{750}	65.75 ± 6.94
Forgetron_{500}	48.58 ± 10.26

Table 6.2: Performance of the global multi-classifiers on the question classification task. The multi-classifiers are created by combining the six binary classifiers whose performance is shown in Table 6.1. All models are fixed for testing. Each test instance is presented to every classifier to predict the instance's inclusion in the given class. The label of the classifier with the largest confidence is selected as the global multi-classifier's prediction for that instance. The performance of the five models trained on different permutations of the data are averaged and presented here.

6.3 Boundary Classification in Semantic Role Labeling

Automated Semantic Role Labeling (SRL) is a task that has received great attention recently. Such systems require the ability to detect the word embodying the predicate as well as the the ability to detect and classify the sequences constituting the predicate's arguments. It has been shown that syntactic information can effectively encode predicate-argument relationships and be leveraged for a machine learning solution, and the Shalow Semantic Tree Kernel [Moschitti, Pighin & Basili, 2008] has proved its ability to do just that.

The task examined herein is to discriminate between nodes which are a part of the predicate's argument (regardless of role) and those which are *NARG* (non-argument) nodes. This Boundary Classifier (BC) is the first step before a Role Multi-classifier (RM) would assign each example determined to be an argument of the predicate the most appropriate label. This technique of splitting

node labelling into two tasks greatly reduces the computational cost as one needs only to apply the BC to all the parse-tree nodes and the RM can ignore irrelevant nodes. This two stage technique was first suggested by Gildea & Jurafsky [2002] and later extended into a four step heirarchical architecture by Moschitti, Giuglea, Coppola & Basili [2005].

The dataset is taken from the CoNLL Shared Task introduced by Carreras & Màrquez [2005] and consists of 4,463,066 examples where the predicate-argument pairs have been manually annotated. Each example includes a syntactic tree as well as linear features generated from automatic parses as detailed in [Moschitti et al., 2008]. Approximately 5% of the corpus are positive examples.

To compare online performance against SVM, we examine the results obtained by Severyn & Moschitti [2010]. They demonstrate an SVM trained on the same 1,000,000 instances and tested against the two final sections of the corpus named *sec23* and *sec24*. We present the F_1 score of the fixed online models against each of these datasets. We made use of the same machines to run our experiments (Intel®Xenon®2.33GHz CPU with 6Gb of RAM running the Linux kernel 2.6.18) so that the computational time may be directly compared.

6.4 Semantic Role Labeling Experimental Results

In Figure 6.1 we see the online performance of the algorithms as they are trained on the dataset. The PA-I algorithm demonstrates the highest level of performance early in training (a), but is matched by the Averaged Perceptron after approximately half of the data (500,000 instances) have been observed (b). The Projectron++ is the next-most performant, followed by the Perceptron and the Forgetron. It is interesting to note how the Forgetron demonstrates a sudden loss of performance about halfway through the data, before slowly recovering to its previous level of performance.

We observe in Figure 6.2 that the growth rate of the Projectron++ algorithm in this case is much greater than in the previous experiments of Chapter 5. This may be due to the high-dimensional nature of the tree kernel coupled with the complexity of the SRL Boundary Classification problem.

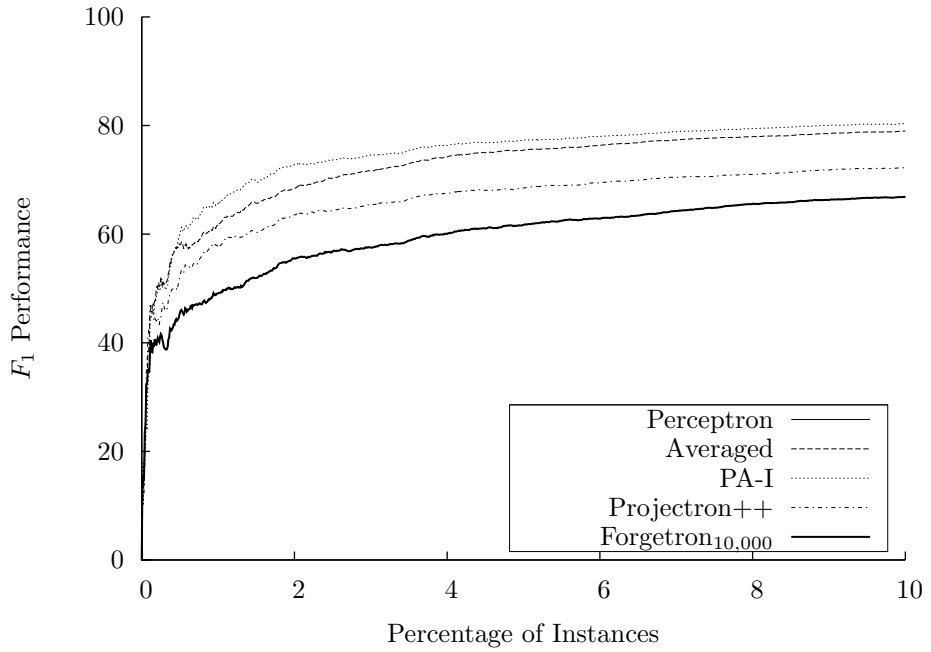
The Averaged Perceptron however, demonstrates a greatly reduced support set size compared to the Perceptron. Another interesting observation is that while initially growing much faster, the PA-I support set slows down to the same rate as the Perceptron after approximately 30% of the instances.

Severyn & Moschitti [2010] observe that training an SVM on the first million instances of SRL task corpora takes approximately 7.5 days. While the Projectron++ took a similar amount of time to complete its training, all the other online algorithms were much faster to train. Their progress can be seen in Figure 6.3, and final training times are listed in Table 6.3.

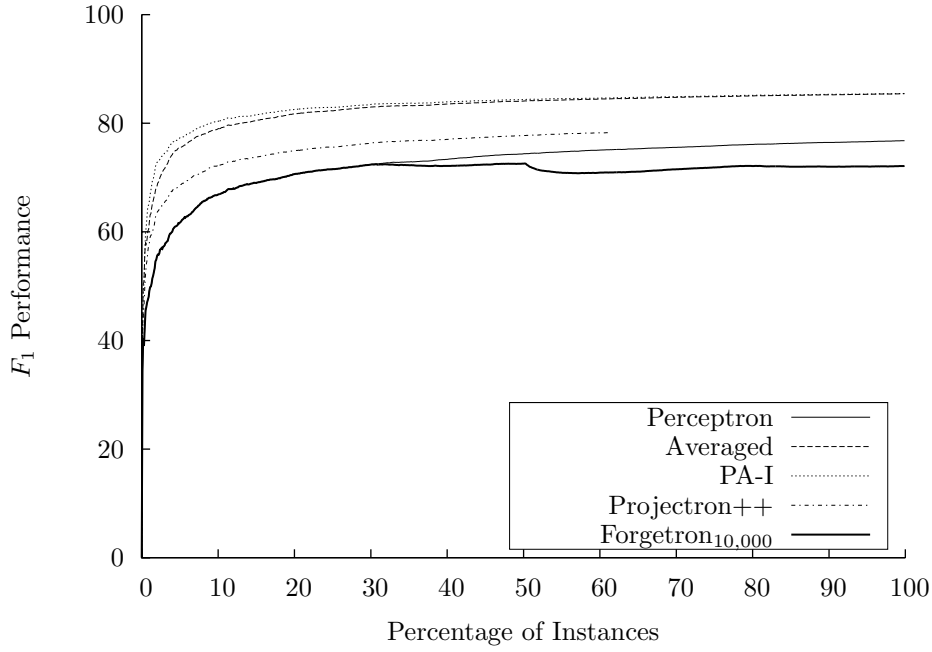
After training on one million instances, the online models were fixed and tested against *sec23* and *sec24* sets, the results of which are shown in Table 6.3. We observe that the online algorithms demonstrate performance that very closely approaches that of the SVM. Particularly successful was the Averaged Perceptron which did so while using less than half the number of support vectors.

Algorithm	Training time	SV	*sec23*	*sec24*
SVM	178.42	61,881	83.82	81.17
Perceptron	16.63	28,334	80.36	78.22
Averaged	15.50	25,559	83.35	81.16
PA-I	42.52	70,412	83.03	81.05
Projectron++	186+	21,552+	–	–
$\text{Forgetron}_{10,000}$	9.97	10,000	68.33	66.53

Table 6.3: F_1 performance of fixed models on two test sets. The SVM performance is quoted from [Severyn & Moschitti, 2010]. At the time of writing, the Projectron++ algorithm had not completed its training. Training time is listed in hours, and SV indicates the number of support vectors in the model.



(a) Online performance on first 10% of instances



(b) Online performance over all instances

Figure 6.1: Online performance on the SRL Boundary Classification task. Where 100% of the Instances corresponds to the set of one million candidate nodes. These learning curves show the cumulative performance of each algorithm as it processes the corpora.

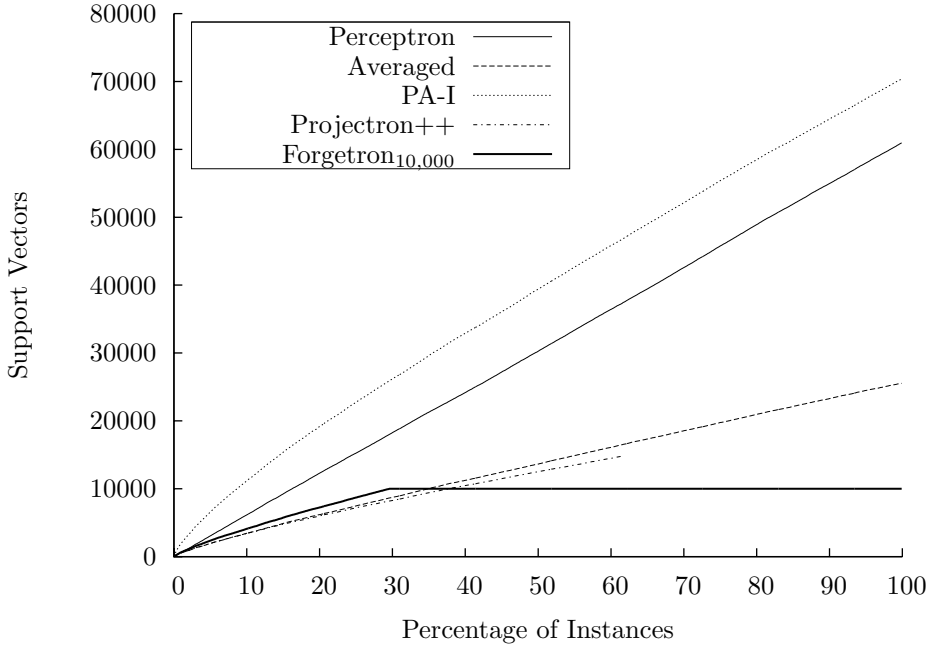


Figure 6.2: Growth of support set during SRL Boundary Classification training, where 100% of the instances corresponds to the set of one million candidate nodes.

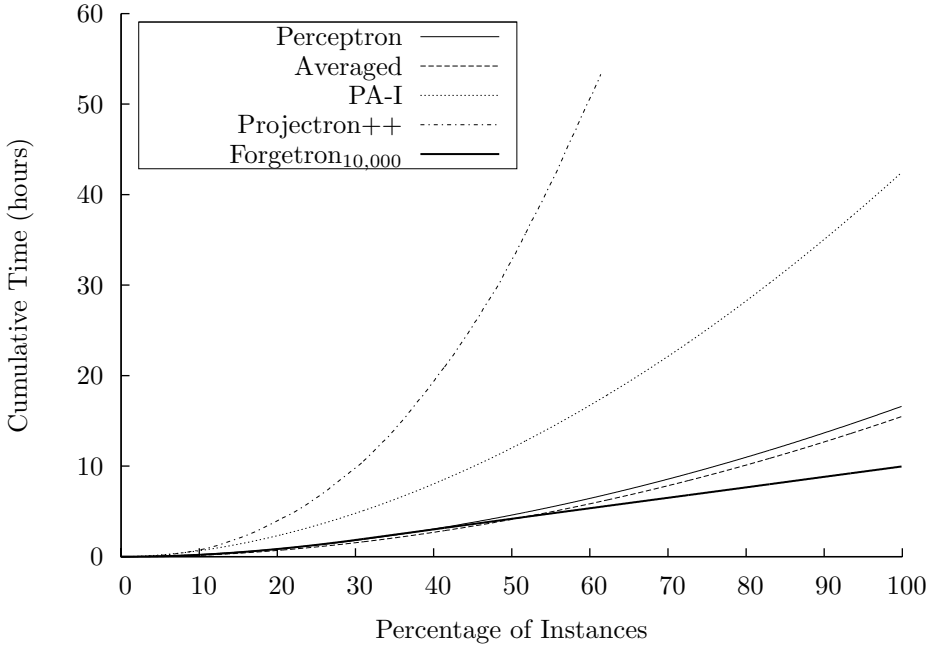


Figure 6.3: Cumulative runtime during SRL Boundary Classification training, where 100% of the instances corresponds to the set of one million candidate nodes.

Chapter 7

Conclusion

> Theoretically-motivated algorithms consistently outperform the heuristic approach [Dekel et al., 2008]

This sentiment is mirrored in the adoption of Tree Kernels over individual feature engineering in natural language processing tasks. Feature sets hand tuned for specific problems cannot hope to match such generally applicable kernels.

7.1 Summary

This thesis has investigated a number of state of the art online learning algorithms for binary classifications, and examined their performance in a number of situations.

While the Support Vector Machine demonstrates clearly superior accuracy in almost all cases, these experiments show the viability of obtaining near state-of-the-art performance using single-pass online learning algorithms. These algorithms are able to be trained using a single pass through the training data and as such are typically much faster to train than a Support Vector Machine for the same problem. In fact, on the Semantic Role Labeling Boundary Classification task the Averaged Perceptron was able to match the SVMs performance with one third the number of support vectors and one tenth the cpu-runtime during training.

Another important caveat is that when dealing with data with an unbalanced number of positive and negative examples, SVMs require a tuned value for their cost-factor hyperparameter in order to obtain good performance. While we

attempt to show a fair comparison between the two, the online algorithms did not receive this kind of tuning, but still performed quite well in the face of this kind of data.

The Perceptron, while being one of the oldest linear classifiers demonstrates good performance. The Averaged Perceptron leverages the whole set of models generated during training, giving each a weighted vote proportional to their performance. This technique helps protect against over-fitting, and can be implemented with very little additional overhead. The Passive Aggressive family of algorithms build upon the Perceptron's successes by not only updating when a classification error is made, but also when the confidence of a correct classification is too low. Their approach generates accurate models at the expense of storing many support vectors, especially when dealing with noisy data.

These techniques perform well, but their models can grow unboundedly when presented with a continuous stream of data. Many techniques have arisen to address this shortcoming.

The Forgetron algorithm is a theoretically well founded approach to learning under the constraint of a fixed budget of support vectors, however its accuracy is upper bounded by that of the Perceptron as it discards data while its time and space complexity are much lower. The Projectron algorithm does not heed a fixed budget, but stores new instances as a linear combination of existing ones when possible. This guarantees the size of the model to be finite even in the face of continuous data. The Projectron++ goes a step further by taking the same aggressive update approach as the PA algorithms, giving it accuracy upper bounded by PA while working within a fixed budget.

The maintenance of the inverse Gram matrix ($\mathbf{K^{-1}}$ in Orabona et al. [2008]) means that while the Projectron algorithm stores far fewer support vectors it still requires a long computational time.

The Projectron derives its savings from ensuring a small set of support vectors. However, in the Semantic Role Labeling Boundary Classification experiments it was observed that its support set grew much faster than in the other situations, and as such resulted in very long processing times. Further investigation into the causes behind this are required to fully evaluate the Projectron's effectiveness when applied to problems using structured kernels.

The results found herein suggest that high-accuracy online learning algorithms are possible and that they can serve some purposes that are not well suited to SVMs, such as when presented with extremely large datasets. These algorithms and techniques have proven themselves not only in traditional feature-

vector kernel spaces, but also when using extremely high-dimensional structural kernels. This observation is heartening as many problems in the natural language processing domain are well described by annotated parse trees, and often involve massive amounts of data which can pose a problem for support vector machines.

7.2 Critical Analysis

One of the main drawbacks of the presented empirical evaluation was in the choices of hyperparameters. In the face of unbalanced ratios of positive and negative examples in the dataset, it is sometimes necessary to choose an appropriate value for the cost-factor when training an SVM in order to obtain non-trivial results. For the Adult dataset a very rough search was used to find an acceptable value. However, similar tuning was not performed for the online algorithms.

In fact, the hyperparameters for the online algorithms were chosen based on suggestions in the literature and not changed. This was particularly troublesome for the Projectron++ algorithm when presented with the SRL dataset. Using a fixed value of $\eta = 0.1$ resulted in a very dense model and very long training time.

Orabona et al. describe a method where η can be set dynamically throughout training. As the model accumulates support vectors and approaches a set budget, the value of η allows fewer instances to be added to the model. This technique is described in their paper as an attempt to make the Projectron algorithm comparable with the Forgetron and it's fixed budget. At the beginning of each round η is set to be

$$\eta = \frac{1}{2U}\left(2\ell(f_{t-1}(\mathbf{x}_t), y_t) - \|P_{t-1}k(\mathbf{x}_t, \cdot)\|^2 - 0.5\right),$$

where the value of U is from the derivation of the Forgetron by Dekel et al. [2008]. Given a maximum number of support vectors B, we have a maximum value for $U = 1/4\sqrt{(B+1)/\log(B+1)}$.

This might have served the SRL task better, and additional experimentation is needed to verify the Projectron's viability in this setting.

The value of the aggressiveness parameter C for the Passive Aggressive algorithm was similarly set using recommendations from the literature. However, future work is needed to compare the effects of selecting different values for C against the other online algorithms.

7.3 Future Work

The online learning framework allows for the creation of unique algorithms with widely varying capabilities and merits further exploration. The rest of this section describes some of the possible avenues of research that are outside the scope of this thesis.

Forgetful Projectron While the Forgetron reduces the impact of removing an example by gradually lowering its effect on the hypothesis over time, by the time an instance is finally removed it is already largely forgotten. The Projectron on the other hand, reduces the impact of removal by attempting to store the information that would be lost as a linear combination of that which remains. While some information can be lost in the projection into the space spanned by the instances in memory, no observations are discarded outright.

It would be interesting to see a combination of the two approaches. Consider the following variant to the Projectron algorithm. Instead of projecting new instances onto the existing set in order to reduce the number of support vectors, the new instance is always added. The projected hypothesis for this variant would be created by removing the oldest stored instance and projecting it onto the remaining support set. While potentially even more computationally expensive as the inverse Gram matrix might not be able to be updated incrementally, this variant might be able to handle datasets that exhibit concept drift while maintaining a finite budget.

Dynamic Aggressiveness The Crammer et al. note that the value of the aggressiveness parameter C is a tradeoff between the learning rate and the final model performance. When C is small, the algorithm shows a slower progress rate but ultimately reaches a higher level of performance. For large values of C the opposite is true. They also note that "as a general rule-of-thumb, C should be small when the data is noisy."

It would be interesting to attempt to let the algorithm itself set the value of C dynamically throughout learning. While early noisy examples may be added to the support set reducing the final accuracy, techniques for maintaining a fixed budget proposed by Wang & Vucetic [2010] could be adapted to weed them out. This could be considered a kind of Passive Aggressive boosting technique.

Appendix A

SVM-Light-TK Usage

To specify the use of online algorithms, set the flag *-z o*.

```
svm_learn -z o -A perceptron datafile
```

To specify which online algorithm to use, set the *-A* flag followed by one of the following options:

```
perceptron
averaged
pa
pai
paii
forgetron
tunedforgetron
greedyforgetron
projectron
projectron++
```

To specify the kernel to use, set the *-t* flag.

```
0: linear (default)
1: polynomial (s a*b+c)^d
2: radial basis function exp(-gamma ||a-b||^2)
3: sigmoid tanh(s a*b + c)
4: user defined kernel from kernel.h
5: combination of forest and vector sets according to W, V, S, C options
```

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